Fundamentals of Boundary Layers

**Flow of inviscid Flow using Velocity Potential**

Considering a situation where effect of viscosity is negligible.

Re >> 1, very high Reynold’s number or fluid flow

Flow over a submerged body, whe whole flow field can be determined using potential flow theory and boundary layer theory.

1. Potential flow: Irrotational flow, 2-D (planar) flow; Incompressible flow; Viscous forces negligible

Equation of Motion for Inviscid Flow

ρD**v**/Dt = -▽p + ρ**g**

Assumptions:

1. Incompressible flow
   1. ρ = constant
   2. ▽**v** = 0
2. Irrotational flow
   1. ▽ ✕ **v** = 0
   2. Two-dimensional, steady flow)

ρD**v**/Dt = -▽p + ρ**g**

**v** ▽**v**  = 1/2 ▽**v**2 - **v** ✕ (▽ ✕ **v**)]

ρ(1/2 ▽**v**2) + P = constant

ρ(1/2 **vx**2+ 1/2 **vy**2) + P = constant

To solve for velocity and pressure profile, we need additional equations.

∂vx/∂x + ∂vy/∂y = 0

∂vx/∂y + ∂vy/∂x = 0

**v** = -▽Φ

Why we picked it

▽(-▽Φ) = ▽2Φ = 0

Φ(x,y)

vx = ∂Φ/∂x; vy = ∂Φ/∂y

Ψ(x,y)

vx = ∂Ψ/∂y; vy = ∂Ψ/∂x

Complex velocity

w(z) = Φ(x,y) + iΨ(x,y)

Z = x + iy

Example 4.3-1

w(z) = -v∞R[z/R + R/z]

Find stream and potential functions

w(z) = Φ(x,y) + iΨ(x,y)

1. To separate real and imaginary parts,

Z = x + iy

w(z) = -v∞R[z/R + R/z] = -v∞[z + R2/z] = -v∞[(x+yi + R2/(x + yi)]

w(z) = -v∞[(x+yi) + R2(x-yi)/(x2+y2)] = -v∞[x + r2x/(x2+y2) + i(y - R2y/(x2+y2)]

Φ(x,y) = -v∞[x + r2x/(x2+y2)]

Ψ(x,y) = -v∞[(y - R2y/(x2+y2)]

Shell momentum balance (unidirectional, dependent on only one direction)

Can also use NVS

Unsteady state (semi-infinite fluid or parallel plate)

Challenge: type of equation - combination/separation of variables

One-dimensional flow (dependence on multiple directions)

Channel flow (vx(z,y), w >> h not true)

Multi Dimensional flow (submerged object)

Velocity potential

1. Finding the velocity components near the cylinder

dw/dz = -vx + ivy

W = -v∞(z + R2/z)

dw/dz = -v∞(1 - R2/z2)

Z = reiθ

Z2 = r2ei2θ

dw/dz = -v∞(1 - R2/r2ei2θ)

dw/dz = -v∞(1 - R2/r2(cos(2θ) + isin(2θ)

dw/dz = -v∞(1 - R2(cos(2θ) + isin(2θ))/r2(cos2(2θ) + isin2(2θ))

dw/dz = -v∞(1 - R2(cos(2θ) + isin(2θ))/r2)

vx = -v∞(1 - R2cos(2θ)/r2)

Vy = -v∞(1 - R2(sin(2θ))/r2)

On surface of cylinder, r = R

Vx = -v∞(1 - cos(2θ))

Vy = -v∞(1 - sin(2θ))

1. Getting pressure distribution

Bernoulli's equation: 1/2 ρ(vx2 + vy2) + P = constant

Far from the cylinder, v = v∞

1/2 ρ(v∞2) + P∞ = constant

On the surface of the cylinder

1/2 ρ(vx2 + vy2) + P = constant

1/2 ρ(v∞2(1-cos(2θ))2 + vy2sin2(2θ)) + P = 1/2 ρ(v∞2) + P∞

cos(2θ) = 1 - 2sin2θ

sin(2θ) = 2sin(θ)cos(θ)

vx2 + vy2 = 4v∞2sin2θ + P = 1/2 ρ(v∞2) + P∞

P - P∞ = 1/2 ρ(v∞2) - 4v∞2sin2θ